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1234 to the 23^{rd}

Name ________________________________ Date _____________

The Problem

$(1{,}234)^2$ means 1,234 * 1,234; $(1{,}234)^3$ means 1,234 * 1,234 * 1,234; and so forth. When $(1{,}234)^{23}$ is completely multiplied out, what will the number be in the ones place?

1234 to the 23rd (continued)

<u>Solution Strategy: Make a Simpler Problem</u>

It is very difficult to think about a number as large as this. Let's make a simpler problem and look for a pattern to help us solve the larger problem.

Since the problem is asking about the ones place, let's look only at the ones place for right now. The number in the ones place is 4. This means that each time we raise the number to the next power, we are multiplying the number in the ones place by 4. Let's see what that looks like for the exponents 2 through 6.

Exponent (power)	Number in Ones Place	Number Model	New Number in Ones Place
2	4	4 x 4 = 16	6
3	6	4 x 6 = 24	4
4	4	4 x 4 = 16	6
5	6	4 x 6 = 24	4
6	4	4 x 4 = 16	6

We are beginning to see some patterns. First of all, we can see that the number in the ones place alternates between 6 and 4. We also see that when the power is even, the number in the ones place is 6. When the power is odd, the number in the ones place is 4.

How can we extend this information to answer the larger problem? First of all we look at the exponent in question. It is 23. We ask ourselves, "Is it even or odd?" It is odd. We know from our pattern analysis for the exponents 2 through 6 that when the power is odd, the number in the ones place is 4. This is the information we are looking for!

<u>ANSWER: When $(1,234)^{23}$ is completely multiplied out, the number in the ones place is 4.</u>

2 Trains Meet

Name ______________________________ Date _______________

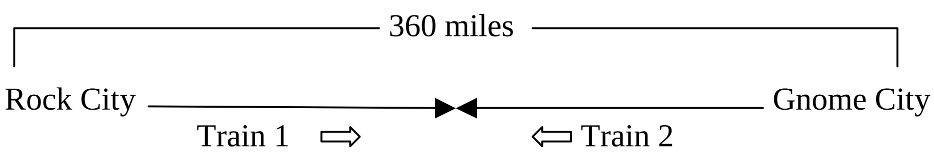
The Problem

A train leaves Rock City at an average speed of 50 miles per hour and heads for Gnome City. Another train leaves Gnome City at an average speed of 40 miles per hour and heads for Rock City. If the route is 360 miles long, how many hours will it take for the 2 trains to meet?

2 Trains Meet (continued)

<u>Solution Strategy: Draw a Picture/Guess and Check</u>

Let's first draw a picture to make the problem a little clearer.



Look at the picture carefully. We notice from the picture that the distance traveled by Train 1 up until the moment the 2 trains meet added to the distance traveled by Train 2 up until the moment the 2 trains meet is equal to the total distance between the 2 cities.

Now let's make some guesses about the solution. Let's say that the 2 trains would meet after 2 hours. Let's try it.

> Train 1 is traveling at 50 mph → for 2 hours → that's 50 x 2 or 100 miles
> Train 2 is traveling at 40 mph → for 2 hours → that's 40 x 2 or 80 miles
> 100 miles + 80 miles = 180 miles

No. We were told that the distance is 360 miles. Our guess is about ½ as big as it needs to be.

Let's try 4 hours.

> Train 1 is traveling at 50 mph → for 4 hours → that's 50 x 4 or 200 miles
> Train 2 is traveling at 40 mph → for 4 hours → that's 40 x 4 or 160 miles
> 200 miles + 160 miles = 360 miles

That's it!

<u>***ANSWER: It will take 4 hours for the 2 trains to meet.***</u>

ABCDEF Equations

Name ______________________________________ Date ________________

The Problem

Solve for the variables A through F in the equations below, using the digits from 0 through 5. Every digit should be used only once. A variable has the same value everywhere it occurs, and no other variable will have that value.

$$A + A + A = A^2$$
$$B + C = B$$
$$D * E = D$$
$$A - E = B$$
$$B^2 = D$$
$$D + E = F$$

ABCDEF Equations (continued)

<u>Solution Strategy: Look for a Pattern</u>

Let's first look at the equations and see if we recognize any of the patterns. As we look through the list, we see the equation ***B + C = B***. We know that when 0 is added to a number, it stays the same. Therefore, C must be equal to 0. **<u>C = 0</u>**

Let's look at the other equations for a pattern we recognize. We see D * E = D. We know that when a number is multiplied by 1, it stays the same. E = 1

<u>Solution Strategy: Guess and Check</u>

Are there any other equations that have familiar patterns or combinations? None really stand out. So we need to try another strategy. Let's make some guesses at values and try them out. The equation with all ***A's*** is interesting. What digit tripled is the same as that digit squared? We know that it must be a relatively small number because when you start squaring numbers, they get large fairly quickly. So let's focus on small numbers. We've already used 0 and 1.

<u>Let's try 2.</u>
2 + 2 + 2 = 6 Does $2^2 = 6$? NO, it's 4.

<u>That wasn't it. Let's try 3.</u>
3 + 3 + 3 = 9 Does $3^2 = 9$? YES, that's it! *A = 3*

We now know the values for A, C, and E. We see that there is an equation with 2 of the values, ***A – C = B***. Let's substitute in the values for A and C (3 – 1 = 2). **<u>B = 2</u>**
Now that we know the value for B, we can solve the equation, ***B² = D*** ($2^2 = 4$). **<u>D = 4</u>**

Now that we've solved for D, we can substitute the values for D and E into the equation ***D + E = F*** and solve for F (4 + 1 = 5). **<u>F = 5</u>** (We might also notice that there is just one variable left and one digit left, so F has to be 5!) Now we have solved for all of the variables.

<u>ANSWER: The values for the variables A through F are as follows:</u>
<u>A = 3, B = 2, C = 0, D = 4, E = 1, F = 5.</u>

Alicia's Babysitting Job

Name ____ Date ________________

The Problem

Alicia was paid $125 for babysitting five days after school for the Smith family. Each day Mrs. Smith paid her $3 more than the day before. How much money did she earn on the first day?

Alicia's Babysitting Job

Solution Strategy: Guess and Check

We know that Alicia earns $125 for the entire week and that she earns $3 more each day. Let's make a random guess about what we think she made on the first day. Let's say she made $20 on the first day. Her earnings for the five days would look like this:

$20 + $23 + $26 + $29 + $32 = $130 ***That's too high.***

Now that we have made a guess, we can examine our results and adjust our next guess to come even closer. We were too high with our first guess, so let's try something lower than $20. Let's try $15. If she made $15 on the first day, her earnings for the five days would look like this:

$15 + $18 + $21 + $24 + $27 = $105 ***That's too low.***

We see that the actual value lies somewhere between $15 and $20. Our first guess was only off by $5 (too high). Our second guess was off by $20 (too low). So let's make a guess that's closer to our first guess but a little lower. Let's try $19. Her earnings for the five days would look like this:

$19 + $22 + $25 + $28 + $31 = $125 ***That's it!***

ANSWER: Alicia earned $19 on the first day.

Angelina's Club

Name ______________________________________ Date ________________

The Problem

Angelina and her friends formed a club named the Extremely Cool Club. They wanted to assign a unique 4-digit secret code number to each member of the club. They decided to use the digits 1, 3, 7, and 9 for their numbering system and each of these digits can appear only once in every secret code number (i.e. 1379 is a valid number, but 1133 is **not** a valid number). What is the maximum number of members who could join the club if everyone is to be assigned a unique secret code number?

Angelina's Club (continued)

<u>Solution Strategy: Make a List</u>

Let's make a list to show all the possible combinations of 4-digit numbers that can be made from the digits 1, 3, 7, and 9.

1379
1397
1739
1793
1937
1973
3179
3197
3719
3791
3917
3971
7139
7193
7319
7391
7913
7931
9137
9173
9317
9371
9713
9731

<u>ANSWER: There are 24 different unique combinations of 4-digit secret code numbers available for the club. They can have 24 members in their club.</u>

Average of a List of 7 Numbers

Name ____ Date ________________

The Problem

You have a list of 7 numbers. The average of the numbers is 9. If you take away one of the numbers, the average of the numbers is 8. What number did you take away?

Average of a List of 7 Numbers (continued)

<u>Solution Strategy: Draw a Diagram</u>

First, let's think about what we know about the term ***average***. We know that when we are finding the average of a group of numbers, we add them up and find their sum, and then we divide that sum by how many numbers we have in the group. So if we are told that we have 7 numbers in the group and the average is 9, what do we know about the sum of the numbers in the group? Let's draw a rate diagram.

Units	Sum of all the numbers in the group	How many numbers in the group?	The average of the numbers in the group
Numbers	?	7	9

<u>63</u> ÷ 7 = 9

We find that the sum of the group of 7 numbers is 63. Now that we see this relationship, we can look at the next part of the problem. If one of the numbers is taken away – we had 7 numbers and now we have 6 – the average is 8. Now we are looking at a group of 6 numbers with an average of 8. Again, let's draw a rate diagram.

Units	Sum of all the numbers in the group	How many numbers in the group?	The average of the numbers in the group
Numbers	?	6	8

<u>48</u> ÷ 6 = 8

Now we know what the sum of the numbers were for each case in our problem, but have we solved the problem? No. We are trying to determine ***what number was taken away*** from the first group, to get an average of 8 in the second group. Therefore, we need to take our 2 sums and find the difference.

63 – 48 = <u>15</u>

The difference between those 2 sums is 15. That means that the number removed from the first group of numbers was 15.

<u>ANSWER: The number removed from the list of 7 numbers was 15.</u>

Basketball Bounce

Name ______________________________________ Date ________________

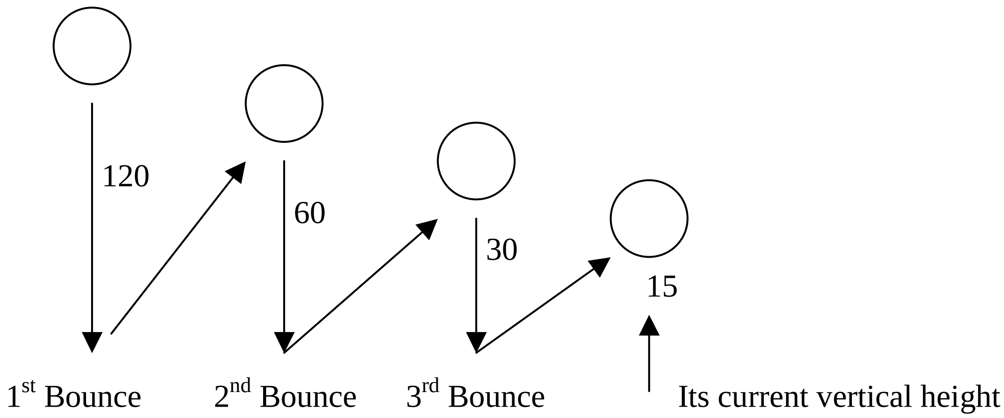
The Problem

A basketball bounces back up about ½ the height from which it is dropped. If a basketball is dropped from 120 feet and keeps bouncing, what is its vertical height after it hits the floor for the 3rd time?

Basketball Bounce (continued)

<u>Solution Strategy: Make a Diagram</u>

This problem is difficult to picture.   Let's draw it and see if that makes it clearer.



We know that each time the ball drops, it comes back up to ½ the height. Starting at 120, we take ½ and get 60, then ½ of that is 30, and finally after the 3rd bounce, it's at ½ of 30 or 15 feet.

<u>ANSWER: The vertical height of the ball after hitting the floor for the third time is 15 feet.</u>

Best Athlete of the Day

Name ______________________________________ Date ________________

The Problem

Bakersfield Elementary School has a field day at the end of the school year in which students compete in sporting events. A group of 5 friends wanted to determine who was the "best" athlete of the day based on the results of 3 events – the 50-yard dash, the mile run, and the rope climb. The chart below shows their results in the 3 events. Each of the events is of equal importance. Tell who you think is the "best" athlete of the day based on the results shown in the chart. Explain why you chose that person.

Names	50-yard Dash	Mile Run	Rope Climb
Brei	8 seconds	6.12 minutes	6 feet
Charlie	10 seconds	8 minutes	7.5 feet
Dani	7.5 seconds	12 minutes	5.8 feet
Eddie	9 seconds	5.59 minutes	4.3 feet
Francine	12.2 seconds	10 minutes	8.2 feet

Best Athlete of the Day (continued)

Solution Strategy: Make a Simpler Problem

Let's take another look at the chart:

Names	50-yard Dash	Mile Run	Rope Climb
Brei	8 seconds	6.12 minutes	6 feet
Charlie	10 seconds	8 minutes	7.5 feet
Dani	7.5 seconds	12 minutes	5.8 feet
Eddie	9 seconds	5.59 minutes	4.3 feet
Francine	12.2 seconds	10 minutes	8.2 feet

It's difficult to make comparisons between the 3 events because they are measured in different units. The 50-yard dash is measured in seconds; the mile run is measured in minutes; and the rope climb is measured in feet. It is also important to note that the ***lower values are the better scores*** in the 50-yard dash and the mile run, but the ***higher values are the better scores*** in the rope climb. How can we make the problem simpler?

Let's assign a number to each person that shows their ranking in each event. Since there are 5 people, let's rank them 1 – 5 in each event with a 5 being the best score and a 1 being the worst score. Our chart would look like this:

Names	50-yard Dash Ranking	Mile Run Ranking	Rope Climb Ranking	Total Score
Brei	4	4	3	11
Charlie	2	3	4	9
Dani	5	1	2	8
Eddie	3	5	1	9
Francine	1	2	5	8

We see that Brei has a score of 11 which is the highest score of the five friends. Therefore, we would choose Brei as the overall "best" athlete for these 3 events.

ANSWER: Brei is the best athlete of the day.

Bikes & Trikes

Name ______________________________ Date ____________

The Problem

There are a total of 12 bicycles and tricycles at the park. Together they have a total of 29 wheels. How many are bicycles and how many are tricycles?

Bikes & Trikes (continued)

<u>Solution Strategy: Guess and Check</u>

We know that bicycles have 2 wheels and tricycles have 3 wheels. We know that there are a total of 12 bicycles and tricycles at the park. Let's make some guesses.

Let's say there are 6 bicycles and 6 tricycles.

6 * 3 = 18 wheels
6 * 2 = 12 wheels

18 + 12 = ______30 wheels______

No, that's too high. But only by 1. So we know that we are close. Let's try adjusting the numbers just slightly. Let's say there are 7 tricycles and 5 bicycles.

7 * 3 = 21 wheels
5 * 2 = 10 wheels

21 + 10 = ______31 wheels____

No. This is even higher. It looks like we adjusted our guess the wrong way. Let's try 7 bicycles and 5 tricycles.

7 * 2 = 14 wheels
5 * 3 = 15 wheels

14 + 15 = ____29 wheels____

That's it! We have found the combination of 12 bicycles and tricycles that gives us 29 wheels. It is 7 bicycles and 5 tricycles.

<u>ANSWER: There are 7 bicycles and 5 tricycles at the park.</u>

Bobby's Mariners Tickets

Name ______________________________________ Date ________________

The Problem

Bobby has 4 tickets for the Mariners game. He invites 3 friends – Tommy, Larry, and Sam – to go with him. Bobby has the first 2 seats in row 5, and the first 2 seats in row 6. The boys are trying to decide on a seating arrangement. How many different combinations of seating arrangements can the boys choose from?

Bobby's Mariners Tickets (continued)

<u>Solution Strategy: Make an Organized List</u>

Let's make a list to show all the possible combinations. It would be hard to keep track if we just started randomly discussing combinations. Let's put our work in an organized list to have a better idea of what possibilities we've tried and what possibilities are still left to try. We'll keep track of how many combinations we find in a column called a running tally.

Row 5, Seat 1	Row 5, Seat 2	Row 6, Seat1	Row 6, Seat 2	Running Tally
<u>Bobby</u>	Tommy	Larry	Sam	1
Bobby	Tommy	Sam	Larry	2
Bobby	Larry	Tommy	Sam	3
Bobby	Larry	Sam	Tommy	4
Bobby	Sam	Tommy	Larry	5
Bobby	Sam	Larry	Tommy	6
<u>Tommy</u>	Bobby	Larry	Sam	7
Tommy	Bobby	Sam	Larry	8
Tommy	Larry	Bobby	Sam	9
Tommy	Larry	Sam	Bobby	10
Tommy	Sam	Larry	Bobby	11
Tommy	Sam	Bobby	Larry	12
<u>Larry</u>	Bobby	Tommy	Sam	13
Larry	Bobby	Sam	Tommy	14
Larry	Tommy	Bobby	Sam	15
Larry	Tommy	Sam	Bobby	16
Larry	Sam	Bobby	Tommy	17
Larry	Sam	Tommy	Bobby	18
<u>Sam</u>	Bobby	Tommy	Larry	19
Sam	Bobby	Larry	Tommy	20
Sam	Tommy	Bobby	Larry	21
Sam	Tommy	Larry	Bobby	22
Sam	Larry	Bobby	Tommy	23
Sam	Larry	Tommy	Bobby	24

<u>ANSWER: There are 24 different combinations of seating arrangements that the boys could choose for the game.</u>

Brei's Long-Distance Phone Call

Name ____ Date ________________

The Problem

Brei likes to call her friend Kiley in California from her home in Washington. Brei's mom makes her pay for all her long-distance phone calls. Last Sunday, Brei called Kiley at 7:00 a.m. and ended the phone conversation at 8:30 a.m. Before 8:00 a.m. on Sundays, it only costs $.35 for the first minute and then $.20 per minute after that to make the call. After 8:00 a.m., the rate goes up to $.40 for the first minute and $.25 per minute after that. How much does Brei owe her mom for the phone call?

Brei's Long-Distance Phone Call (continued)

<u>Solution Strategy: Make a Simpler Problem and Find a Pattern</u>

What do we know about the problem? We know that the phone call started at 7:00 a.m., which is during the "before 8:00" time frame, so ***the first minute cost $.35***. Every minute after that – up until 8:00 a.m. – costs $.20 per minute. Let's look at that info in a table.

Minute	Cost per Minute	Total Cost
1	$.35	$.35
2	$.20	$.35 + $.20 = $.55
3	$.20	$.55 + $.20 = $.75
4	$.20	$.75 + $.20 = $.95

We can see pretty quickly that this is a long, slow process, looking at each minute and adding $.20. Do we see a pattern that might make our job easier? We can calculate how many minutes there are before the "after 8:00" rate kicks in. We have already accounted for the first minute. There are 59 minutes left. Our number model will look like this:

The next 59 minutes = 59 x ($0.20) = $11.80

What about the remaining 30 minutes? Now we have to switch to the higher rate of $.25 per minute. Do we need the information about the first minute after 8:00 and what it costs? No. We have already had our "first minute" on the phone, so it is unnecessary information. Our number model for the remaining 30 minutes would look like this:

The remaining 30 minutes = 30 minutes x ($.25) = $7.50

Now we add the 3 totals together:

The first minute:	$ 0.35
The next 59 minutes:	$11.80
The next 30 minutes:	<u>$ 7.50</u>
Total Cost:	$19.65

<u>ANSWER: Brei owes her mom $19.65 for the phone call.</u>

Checkerboard Problem

Name ____ Date ________________

The Problem

How many squares are there on a standard 8 x 8 checkerboard?

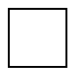
Checkerboard Problem (continued)

<u>Solution Strategy: Make a Simpler Problem</u>

A standard checkerboard has 8 rows and 8 columns of squares. Our first thought may be that there are 8 * 8 or 64 squares. But that is only the number of small squares on the checkerboard. Taking a closer look, we realize that there are many, many different sized squares besides the 1x1 squares. There are 2x2 squares, 3x3 squares, 4x4 squares, etc., not to mention the largest square – the 8x8 square.

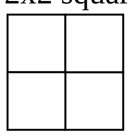
Let's take a look at a simpler problem in order to gain some strategies for solving the larger problem.

Let's take a look at a 1x1 square. How many squares are there in a 1x1 square?



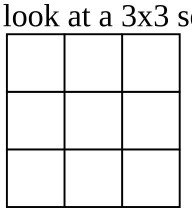
Well, that's easy – there's just one.

Let's take a look at a 2x2 square. How many squares are there in a 2x2 square?



We see that there are 4 smaller squares (1x1) and there is 1 larger square (2x2).
There are a total of (4 + 1) or 5 squares.

Now let's take a look at a 3x3 square. How many squares are there in a 3x3 square?



We see that there are 9 smaller squares (3x3) and there are 4 larger squares (2x2) and there is 1 very large square (3x3). There is a total of (9 + 4 + 1) or 14 squares.

<u>Note to Teacher/Tutor:</u>
You may want to continue with one or two more examples until students begin to see a pattern. Highlighting the squares with different colors may also be helpful to assist students in visualizing all the different squares within the larger square.

Checkerboard Problem (continued)

Solution Strategy: Find a Pattern

Once we have examined our simpler problems, we may begin to see a pattern that will help us solve the larger problem. Let's review what we found:

Size of square	Number of squares found within the larger square
1x1	1
2x2	1 + 4
3x3	1 + 4 + 9
What comes next?	
4x4	1 + 4 + 9 + 16
5x5	1 + 4 + 9 + 16 + 25
6x6	1 + 4 + 9 + 16 + 25 + 36
And so on and so forth…	

Do these numbers look familiar to you? If we look closely, we can see that the pattern is the sum of the perfect squares from 1 to the number that we're working on. Let's look more closely at the pattern for a 6x6 square.

$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2$ = 1 + 4 + 9 + 16 + 25 + 36 = 91 squares

Now that we know the pattern, we can solve for any size square. Our original problem asked the question: How many squares on an 8x8 checkerboard? Using our pattern, we can set up our number model as follows:

$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2$ = 1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 = 204

ANSWER: The number of squares on a checkerboard is 204.

Class President

Name ______________________________________ Date ________________

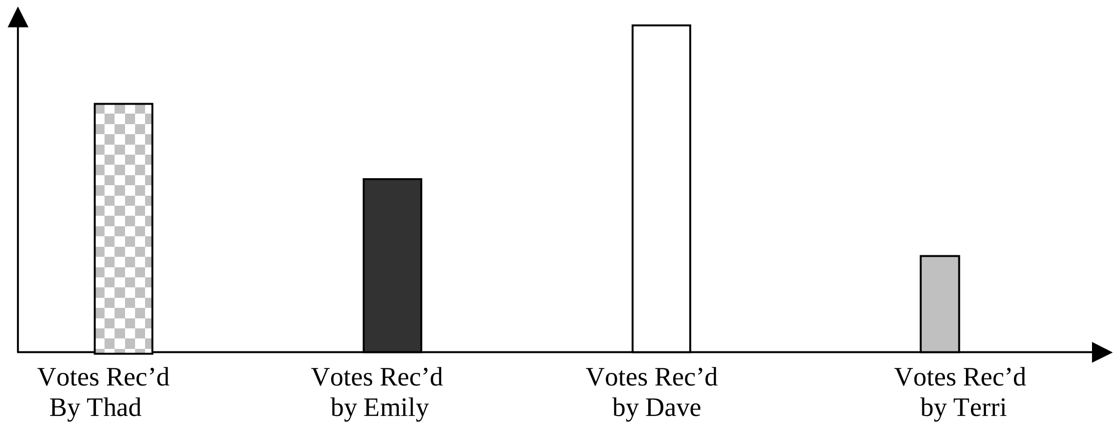
The Problem

Terri, Dave, Emily, and Thad are running for Class President. When the election results came in, Thad received more votes than Emily. Dave received more votes than Thad. Terri received less votes than Emily. Who was elected Class President?

Class President (continued)

<u>Solution Strategy: Make a Graph</u>

Let's put the information into a graph format. Even though we don't know the actual numbers, we can indicate the order of the number of votes received by each student in the election.



Looking at the graph, we see that Dave received the most votes. We don't need to know how many actual votes he received, because we are given clues throughout the problem that tell us the order of the numbers.

<u>ANSWER: Dave was elected Class President.</u>

Comparing Ages of Collin & Anthony

Name ______________________________________ Date ________________

The Problem

Anthony's age plus the cube of Collin's age is 1,739. Collin's age plus the cube of Anthony's age is 1,343. How old are Collin and Anthony?

Comparing Ages of Collin & Anthony (continued)

<u>Solution Strategy: Guess and Check</u>

Let's start with something that's easy to cube. Let's try making Collin 10. 10^3 is 1,000. We are told that Anthony's age plus the cube of Collin's age equals 1,739. 1,739 – 1,000 is 739. This would mean that Anthony is 739 and that's obviously not right. We need to try a larger number, so that subtraction from 1,739 will produce a smaller number.

Let's say that Collin is 11. 11^3 is 1,331. Now we subtract that from 1,739. 1,739 – 1,331 is 408. That's still not right, but closer than our last guess.

Let's say that Collin is 12. 12^3 is 1,728. 1,739 – 1,728 is 11. That's an answer that could be possible. Anthony could be 11 years old.

Let's check it. 11^3 = 1,331. Let's subtract that from 1,343. 1,343 – 1,331 = 12. That's it! Collin is 12 and Anthony is 11.

<u>ANSWER: Collin is 12 and Anthony is 11.</u>

Comparing Weights of a Group of 4th Graders

Name ______________________________________ Date ________________

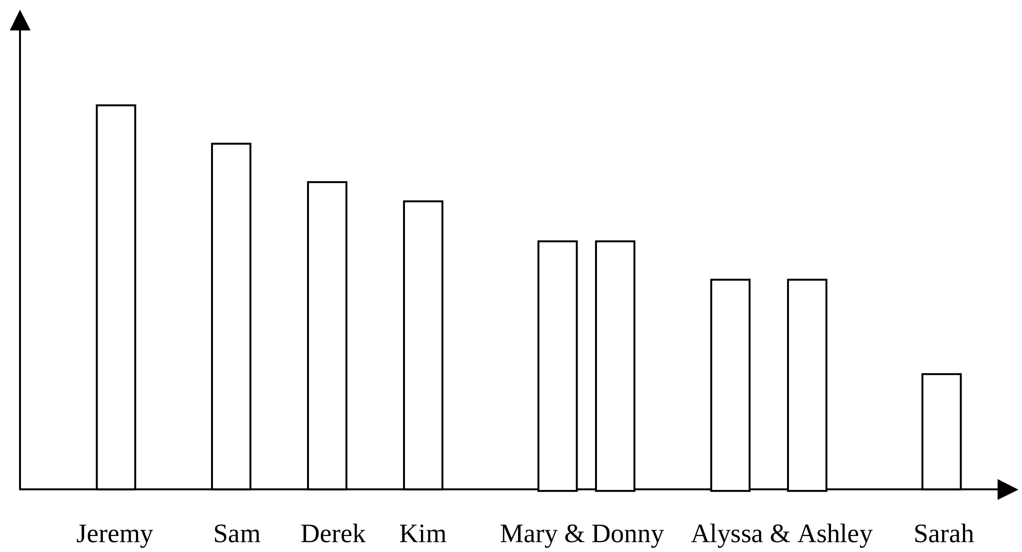
The Problem

A group of 4th graders were comparing their weight. Jeremy weighed more than Sam. Sam weighed more than Kim and Sarah. Mary and Donny weighed the same but less than Kim. Derek weighed more than Kim but less than Sam. Alyssa and Ashley weighed the same, which was less than Mary and Donny. Sarah weighed the least. Who weighed the most?

Comparing Weights of a Group of 4th Graders (continued)

Solution Strategy: Make a Chart

Let's make a bar graph to show the weight comparisons in order to find out who weighs the most. We don't know any of the actual weights of the students, but we do know how they compare to each other.



ANSWER: Jeremy weighs the most.

Courtney's Rectangular Blocks

Name ______________________________________ Date ________________

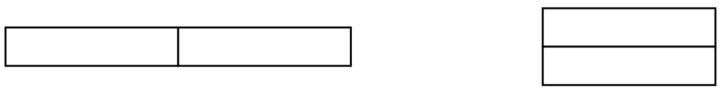
The Problem

Courtney had 36 identical rectangular blocks. Her teacher instructed her to put the blocks together to make larger rectangles so that short sides of the rectangles are only matched up with short sides, and long sides of the rectangles are only matched up with long sides. How many larger rectangles can Courtney make with the 36 blocks?

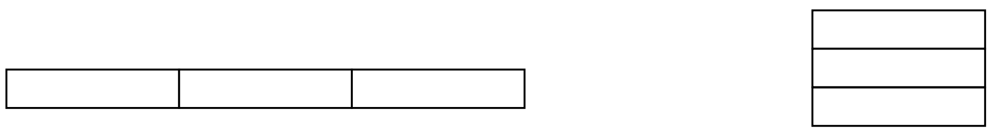
Courtney's Rectangular Blocks (continued)

Solution Strategy: Make a Simpler Problem and Draw a Picture

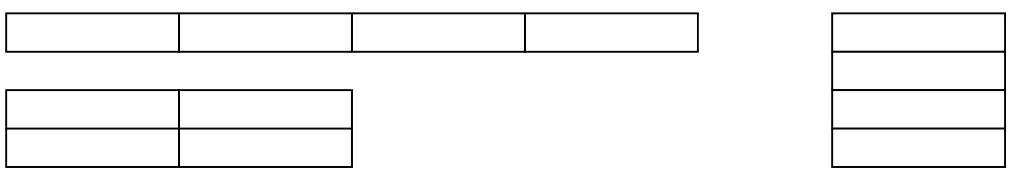
Let's look at just 2 of the rectangles instead of trying to think about all 36. There are 2 ways that we can put the 2 rectangles together; a shape that is 1 x 2 and another shape that is 2 x 1. Let's draw it. (Note that each of the rectangles can be created either horizontally or vertically, but is still considered the same rectangle, rather than 2 different rectangles.)



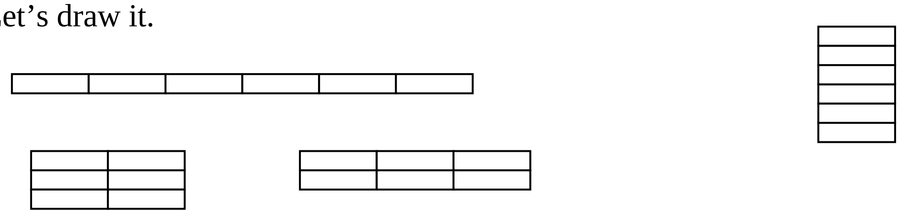
Now let's look at 3 of the rectangles. There are 2 ways to put 3 rectangles together; a shape that is 1 x 3 and a shape that is 3 x 1. Let's draw it.



Now let's look at 4 of the rectangles. There are 3 ways to put 4 rectangles together; a shape that is 1 x 4, 4 x 1, and 2 x 2. Let's draw it.



Let's try 6. There are 4 ways to put 6 rectangles together; 1 x 6, 6 x 1, 2 x 3, and 3 x 2. Let's draw it.



We are starting to see a ***pattern***. Let's switch strategies.

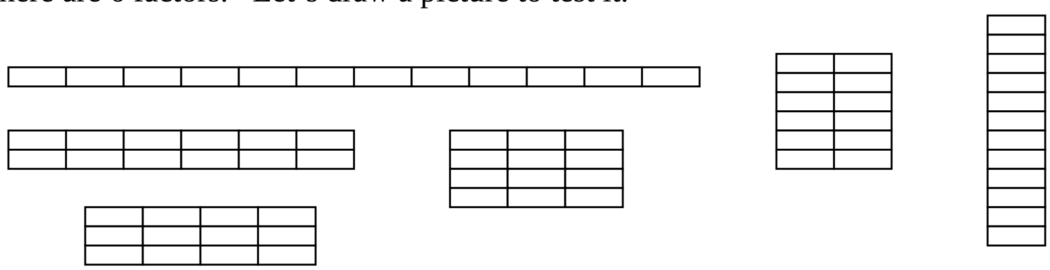
Courtney's Rectangular Blocks (continued)

<u>Solution Strategy: Find a Pattern and Draw a Picture</u>

Let's evaluate the pattern that we are seeing. We see that when a number has 2 factors, there are 2 different ways to make rectangles. For example, 2 has 2 factors (1,2) and there are 2 ways to make rectangles for 2 blocks. 3 also has 2 factors (1,3) and there are 2 ways to make rectangles for 3 blocks. When we tried 4, which has 3 factors (1,2,4), we found that there were 3 different ways to make rectangles for 4 blocks. And finally, when we tried 6, which has 4 factors (1,2,3,6), we found that there were 4 different ways to make larger rectangles for 6 blocks.

The number of rectangles that we can make for each set of blocks seems to be equal to the ***number of factors*** for the given number.

Let's test our theory and randomly select another number from our set (1 – 36). Let's try 12. How many factors are there for 12? The factors of 12 are: 1, 2, 3, 4, 6, and 12. There are 6 factors. Let's draw a picture to test it.



We can now say with reasonable certainty that the number of larger rectangles we can make for ***n*** *rectangular blocks is equal to the number of factors for* ***n****.*

Let's now solve for 36, which is the actual problem we are asked to solve here. The number of factors for 36 is 9. They are 1, 2, 3, 4, 6, 9, 12, 18, and 36.

<u>ANSWER: There are 9 different ways Courtney can place 36 rectangular blocks together to make larger rectangles.</u>

Dice Game

Name ______________________________________ Date ________________

The Problem

Lisa, Rob, and Dave are playing a game with 3 dice. The object is to roll the 3 dice and then use the 3 numerals rolled make as many unique 3-digit numbers as possible. Each numeral can only be used once in the number (unless you roll more than one of a certain number; i.e.if you roll 2 fives, your answer will contain 2 fives). A player scores 10 points for each different 3-digit number that he/she is able to form. In this game, the players rolled the following:

Lisa rolled a 2, 4, and 6.
Rob rolled a 3, 5, and 5.
Dave rolled a 1, 1, and 1.

Who earned the most points in the game?

Dice Game (continued)

Solution Strategy: Make a List

Let's first take a look at Lisa's roll. We need to make a list of all the possible combinations of 3-digit numbers that we can come up with using the numerals 2, 4, 6.

246
264
426
462
624
642

There are 6 possible combinations. Since each 3-digit number is worth 10 points, Lisa earned a total of 60 points for her turn.

Now let's take a look at Rob's roll. We need to make a list of all the possible combinations of 3-digit numbers that we can come up with using the numerals 3, 5, 5.

355
535
553

There are only 3 different combinations, so Rob earned a total of 3 * 10 or 30 points.

Finally, let's take a look at Dave's roll. We need to make a list of all the possible combinations of 3-digit numbers that we can come up with using the numerals 1, 1, 1.

111

There is only one possible combination for 1, 1, and 1. Dave earned 1 * 10 or 10 points. Let's compare. Lisa earned 60 points, Rob earned 30 points, and Dave earned 10 points.

ANSWER: Lisa earned the most points in the game.

Father & Sons Crossing the Lake

Name ______________________________________ Date ________________

The Problem

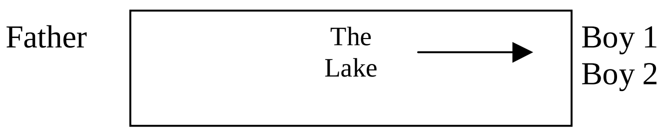
There are 2 boys and their father trying to cross the lake in a boat. Each of the boys weighs 100 pounds, and the father weighs 200 pounds. The boat can only hold 200 pounds at a time. How many trips will it take them to get everyone over to the other side?

Father & Sons Crossing the Lake (continued)

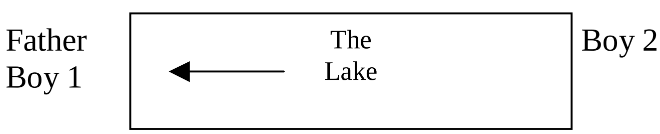
<u>Solution Strategy: Draw a Picture</u>

This problem is difficult because it's hard to picture in our minds. Let's draw a picture of each step to help us see it more clearly.

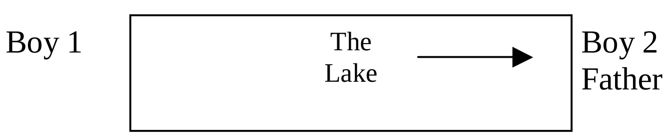
Step 1. We are told that the boat can only hold 200 pounds. The father weighs 200 pounds. Each of the boys weighs 100 pounds. This means we can either have the 2 boys in the boat or the father but never a boy and the father. If the father goes over first, then the 2 boys are stuck on the other side. So, let's start by having both of the boys go over first. That's 1 trip.



Step 2. Now we have 2 boys on one side and the father on the other side. Let's have one of the boys go back to take the boat to the father. That makes 2 trips.

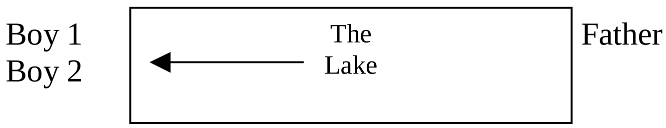


Step 3. Now let's have the father take the boat back over to the other side, leaving one boy on one side and putting the father and the other boy on the other side. That makes 3 trips.

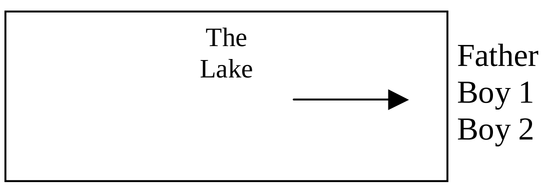


Father & Sons Crossing the Lake (continued)

Step 4. Now let's have the boy who is with the father take the boat back and pick up the other boy. That makes 4 trips.



Step 5. Now both boys can take the boat back to the side where the father is and everyone will be on the same side. That is a total of 5 trips.



<u>ANSWER: It took a total of 5 trips to get everyone over to the other side.</u>

How Many Handshakes

Name ______________________________________ Date ________________

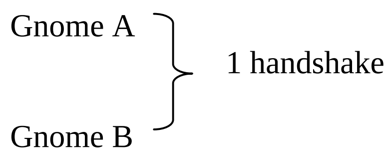
The Problem

At the first meeting of the House of Eccentricities in the government of the Gnomes, each member shook hands with each other member. There are 25 members of the House. How many handshakes took place?

How Many Handshakes (continued)

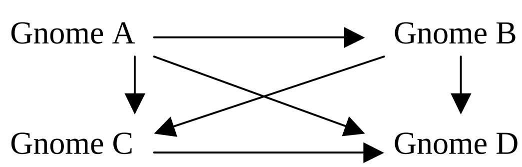
<u>Solution Strategy: Solve a Simpler Problem and Look for a Pattern</u>

It is hard to picture 25 people all shaking hands with each other. Let's look at a simpler problem and see if that will help us look at the harder problem. Suppose there are just 2 gnomes in the House. Let's call them Gnome A and Gnome B.



Suppose there are 3 gnomes in the House. Let's call them Gnome A, Gnome B, and Gnome C.

This results in 3 handshakes. Let's look at 4 gnomes in the House.



This results in 6 handshakes. We're starting to see a pattern.

2 gnomes	1 handshake
3 gnomes	1 + 2 or 3 handshakes
4 gnomes	1 + 2 + 3 or 6 handshakes

Now we can look at the larger problem. If there are 25 gnomes, then our number model will look something like this:

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19 + 20 + 21 + 22 + 23 + 24 = 300

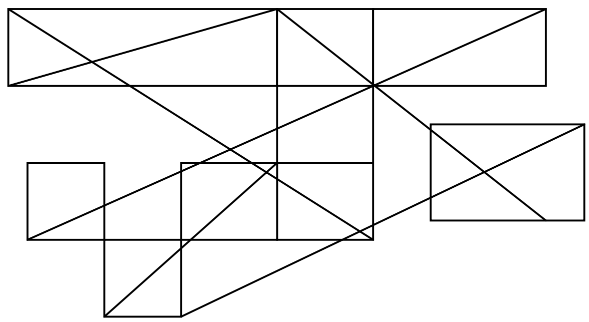
<u>ANSWER: There were 300 handshakes.</u>

How Many Squares & Rectangles?

Name ____ Date _________________

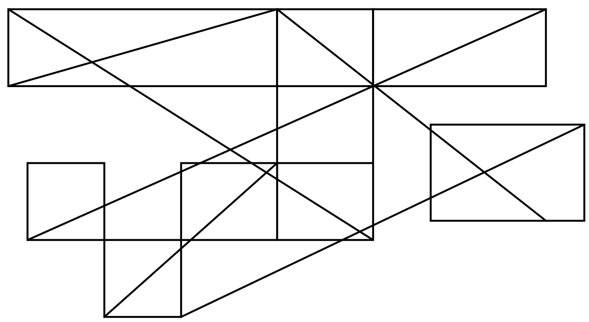
The Problem

If you remove all the diagonal lines, how many different squares and rectangles are there in the figure below?

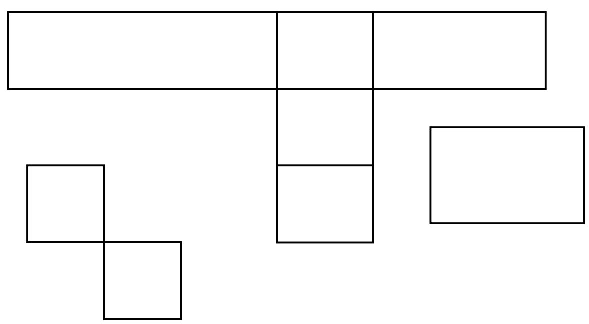


How Many Squares & Rectangles? (continued)

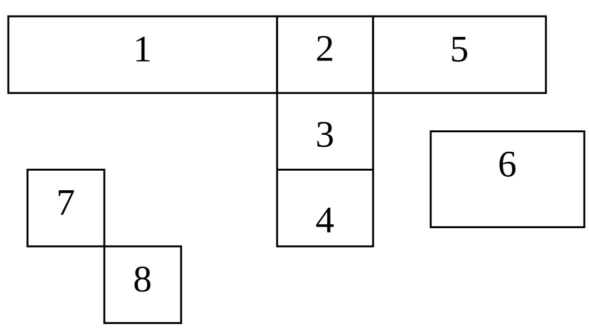
<u>Solution Strategy: Draw a Picture</u>



It's difficult to try and picture all the different squares and rectangles with the diagonal lines on the picture.   Let's redraw the picture without the diagonals.



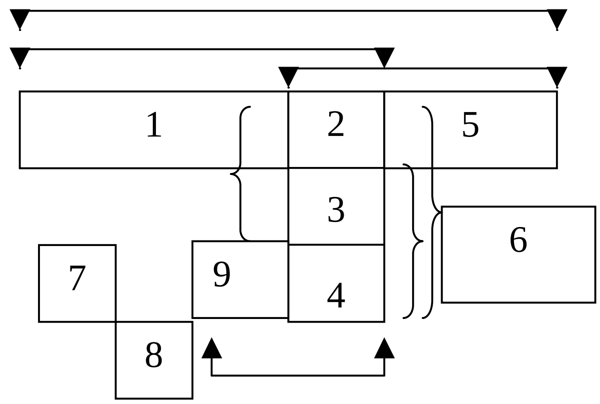
There!   That makes the squares and rectangles much clearer to us. Let's add up all the squares and/or rectangles we see without overlapping any lines.



We see that there are 8 squares/rectangles without overlapping lines.

How Many Squares & Rectangles (continued)

Now let's look for the less obvious ones.



Let's look horizontally first. If we combine 1 and 2, we see there is a rectangle. That makes a total of 9. If we combine 2 and 5, we see another rectangle. Now we're up to 10. If we combine 1, 2 and 5, we have another rectangle. That's 11.

Now let's look vertically. If we combine 2 and 3, we create a rectangle. That's 12. If we combine 3 and 4, there's another rectangle. That's 13. If we combine 2, 3, and 4, we've created yet another rectangle. Now we're at a total of 14. Looking carefully, it appears that we have counted all of the existing rectangles and squares in the diagram.

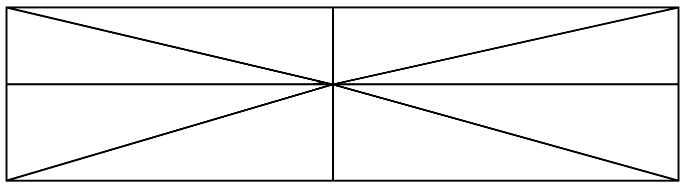
ANSWER: There are a total of 14 squares and/or rectangles in the diagram.

How Many Triangles?

Name ______________________________________ Date ________________

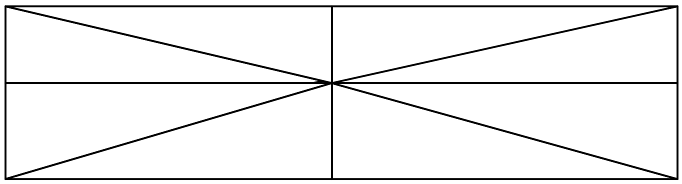
The Problem

How many different triangles can you find in the diagram below?

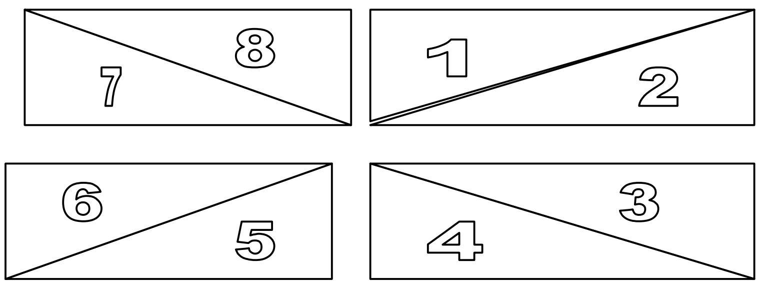


How Many Triangles? (continued)

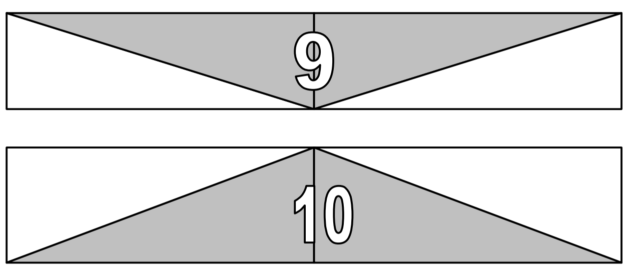
<u>Solution Strategy: Make a Simpler Problem and Draw a Picture</u>



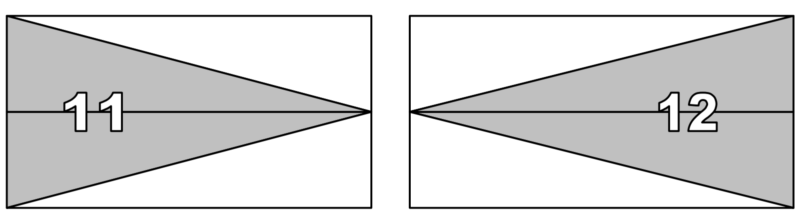
Since it’s difficult to look at the entire diagram at one time, let’s first re-draw the diagram and divide it up into 4 parts and examine each of those parts. We see that there are 2 smaller triangles in each of those 4 parts. That makes 8 triangles.



Now let’s divide the rectangle in half horizontally and see if there are any new triangles that we haven’t already counted. There is one bigger triangle in each half that we haven’t counted yet. Now we are up to 10 triangles.

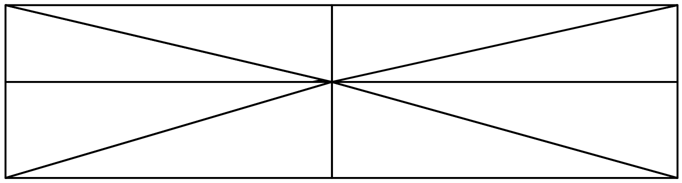


Now let’s divide the rectangle in half vertically and see if there are any new triangles that we haven’t already counted. There is one bigger triangle in each half that we haven’t counted yet. That brings us up to 12.

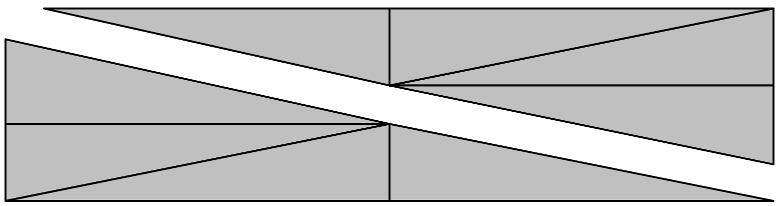


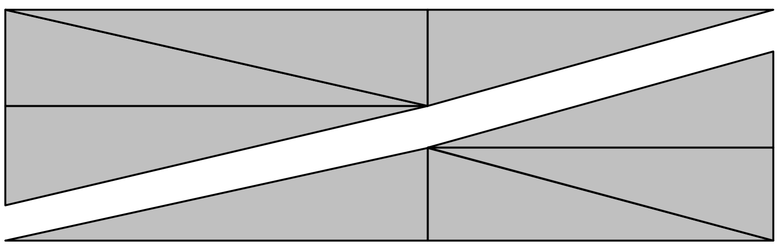
How Many Triangles? (continued)

Now that we’ve examined the parts, we can go back to the original figure and look for the largest triangles.



There are 4 more triangles.





That brings us to a total of 16.

ANSWER: There are 16 triangles in the diagram.

Jeremy's Hondas

Name ______________________________________ Date ________________

The Problem

Jeremy likes Hondas. He owns some cars and some motorcycles. There are 9 Hondas all together. They have a total of 26 tires. How many of the Hondas are motorcycles?

Jeremy's Hondas (continued)

<u>Solution Strategy: Guess and Check</u>

Since there are 9 all together, let's try 2 numbers that add up to 9. Let's try 2 cars and 7 motorcycles.

2 cars with 4 tires each → 2 x 4 = 8 tires
7 motorcycles with 2 tires each → 7 x 2 = 14 tires
8 + 14 = 22 tires

We are close but it's not quite enough.

Let's try 3 and 6.

3 cars with 4 tires each → 3 x 4 = 12 tires
6 motorcycles with 2 tires each → 6 x 2 = 12 tires
12 + 12 = 24 tires

No, but we're getting closer. Let's try 4 and 5.

4 cars with 4 tires each → 4 x 4 = 16 tires
5 motorcycles with 2 tires each → 5 x 2 = 10 tires
16 + 10 = 26 tires

That's it!

<u>ANSWER: Jeremy owns 5 Honda motorcycles.</u>

Magic Number Box

Name ____ Date _________________

The Problem

Fill in the rest of the table by determining the next logical numbers in the pattern.

1	1	3
2	5	13
8	?	?

Magic Number Box (continued)

<u>Solution Strategy: Look for a Pattern</u>

1	1	3
2	5	13
8	?	?

Let's look first across the rows.   Are there any obvious patterns?   **<u>Not really.</u>**

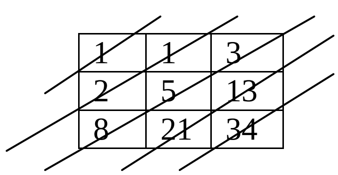
Then let's look down the columns.   Are there any obvious patterns? **<u>Not really.</u>**

Let's look at the diagonals. The number in the upper left box is a 1. That is the only number on the first diagonal.   Let's look at the next diagonal.   There are 2 boxes, and reading from the top box down, they contain the numbers 1 and 2.   Is there a pattern? **<u>Possibly.   1 + 1 = 2.</u>**

Let's go to the next diagonal over.   There are 3 boxes in that diagonal, and reading from top down, they contain a 3, 5, and 8.   Does 1 + 2 = 3 ?   **<u>Yes. The pattern of adding the 2 preceding numbers on the diagonal from top to bottom is still working.</u>**

How about 3 + 5 = 8? **<u>Yes. The pattern is still good</u>**.

Continuing on with this pattern, we see that it holds for **<u>5 + 8 = 13</u>**.
The next number in the sequence will be: **<u>8 + 13 = 21</u>**
The final number in the sequence will be: **<u>13 + 21 = 34</u>**



There! Now the table is complete. (This series of numbers is called the Fibonacci sequence.)

<u>ANSWER: The missing numbers are 21 and 34.</u>

Making Whole Numbers from 2,4,6 and 8

Name ______________________________________ Date ________________

The Problem

How many different whole numbers less than 10,000 can be made from the set of digits {2, 4, 6, 8}? (NOTE: It's okay to use a digit more than once in a number.)

Making Whole Numbers from 2,4,6 and 8 (continued)

Solution Strategy: Make a Simpler Problem and Look for a Pattern

It is difficult to think about all of the whole numbers between 2 and 10,000 with 2, 4, 6, and 8 in them. Let's make a simpler problem and see if we can find a pattern that will help us solve the bigger problem.

Let's first start with the single-digit numbers that use the digits in the given set. Those numbers are:

2, 4, 6, and 8

There are 4. Do we see a pattern? We see that there are as many single-digit whole numbers as there are numbers in the set. Let's continue.

Now let's look at the double-digit numbers that use the 4 digits in our set. Let's create a list of all the possibilities:

22	42	62	82
24	44	64	84
26	46	66	86
28	48	68	88

We see that there are 16 different possibilities in the 2-digit number category. Do we see a pattern? Well, we see that there are 4 x 4 or 16 different possibilities. Is the pattern consistent? Let's look at the triple-digit numbers that use the 4 digits in our set. Here is the list of possibilities:

222	422	622	822
224	424	624	824
226	426	626	826
228	428	628	828
242	442	642	842
244	444	644	844
246	446	646	846
248	448	648	848
262	462	662	862
264	464	664	864
266	466	666	866
268	468	668	868
282	482	682	882
284	484	684	884
286	486	686	886
288	488	488	888

Making Whole Numbers from 2, 4, 6 & 8 (continued)

We see that there are 4 x 4 x 4 or 64 different combinations. Is our pattern remaining consistent? It appears to be. Each time we add another digit, our number of possibilities is 4 times greater.

So the number of combinations for 4-digit numbers would be:
4 x 4 x 4 x 4 = 256

Do we need to check 5-digit numbers? No. The problem only asks us for the numbers below 10,000.

Now we need to add up all of the combinations from each of the simpler categories we looked at:

4 + 16 + 64 + 256 = 340

ANSWER: There are 340 whole numbers that use the numbers in the set {2, 4, 6, 8}and are less than 10,000.

Monica's Square Blocks

Name ______________________________________ Date ________________

The Problem

Monica was given a container of plastic square blocks. She was instructed to make a series of consecutively larger squares by adding blocks each time. The first thing Monica did was put down 1 block to make a 1 x 1 square. Next, Monica added 3 more blocks to make a 2 x 2 square. Then she added 5 more blocks to get a 3 x 3 square, and so on and so forth. What is the size of the square in front of Monica if she has just added 27 blocks?

Monica's Square Blocks (continued)

<u>Solution Strategy: Make a Simpler Problem and Draw a Picture</u>

Let's continue with simpler problems in the same way that Monica is proceeding to see if we can find a pattern that will help us solve the bigger problem. Monica already did the first 2 squares. Her first square was made up of one block (1). Then she added 3 more blocks to get a 2 x 2 square or a total of 4 blocks (1 + 3 = 4). What will come next? Let's draw a picture of what we have so far.

How many blocks do we need to add to get the next bigger square? Again, let's draw a picture to help us see it more clearly.

		1
		2
5	4	3

We see that we need to add 5 blocks to make the next square. We can also see that we have a 3 x 3 square or a total of 9 blocks all together (1 + 3 + 5 = 9). Monica would write down 5 for the number of blocks added and 9 for the total number of blocks.

What will come next? Again, let's draw it to help us visualize it.

			1
			2
			3
7	6	5	4

We see that we need to add 7 blocks to make the next square. We can also see that we have a 4 x 4 square or a total of 16 blocks (1 + 3 + 5 + 7 = 16). Monica would write down 7 for the number of blocks added and 16 for the total number of blocks.

Monica's Square Blocks (continued)

Solution Strategy: Look for a Pattern

Are we starting to see a pattern? Examining our results carefully, we see that there is a pattern in the number we add each time. We are adding ***the next odd number***. We started with 1, then we added 3, then we added 5, and then we added 7.

Is there a pattern in the total number of blocks? There are actually a couple of patterns there. The total is equal to the sum of the numbers added each time or ***the sum of the first 4 consecutive odd numbers*** (1 + 3 + 5 + 7 = 16). We might also observe that the total for each bigger square is equal to ***the first 4 consecutive perfect squares***: 1^2 or 1, 2^2 or 4, 3^2 or 9, 4^2 or 16.

Can we apply these observations to the larger problem we are trying to solve here? ***That is, what is the total number of blocks in the larger square created by adding 27 blocks to the previous one?*** Let's try the next example in our sequence of easier problems and make a prediction about the results using our observations about the pattern so far to see if they hold true. From our observations, we would predict that the next number of blocks to be added would be 9 (the next odd number after 7), and we would guess that the total would be 25 (5^2 = 25 or 1 + 3 + 5 + 7 + 9 = 25). Let's draw it and see if our prediction is good.

				1
				2
				3
				4
9	8	7	6	5

Yes! Our prediction was good. We see that there were 9 blocks added and we now have a 5x5 square or a total of 25 blocks.

Now we can make our prediction about the square that Monica is currently working on. She just added 27 blocks. We can add the odd numbers from 1 to 27 to determine the total blocks: 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 + 27 = 196. We can also look at our pattern with perfect squares. We see that 27 is the 14^{th} odd number in the series of numbers and 14^2 = 14 x 14 = 196. The total is 196.

<u>ANSWER: The square that Monica is currently working on is made up of 196 blocks.</u>

Mr. Mayer's Shirts & Ties

Name ______________________________________ Date ________________

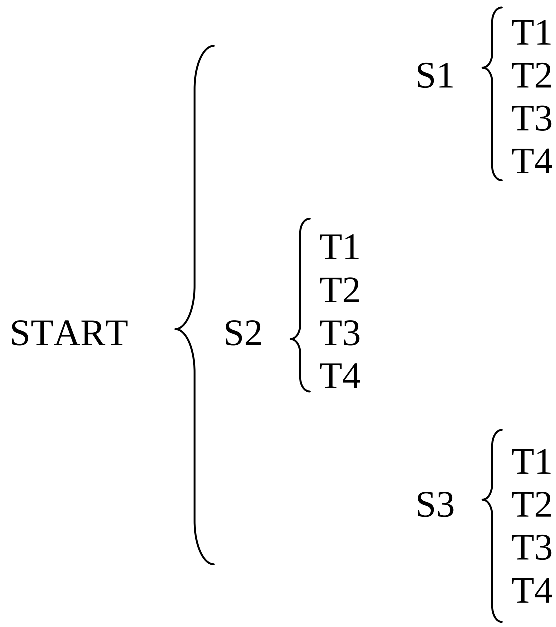
The Problem

Mr. Mayer has 3 shirts and 4 ties. How many different combinations of shirts and ties can he make?

Mr. Mayer's Shirts & Ties (continued)

<u>Solution Strategy: Make a Diagram</u>

Let's make a "tree diagram". We'll label his 3 shirts S1, S2, and S3. We'll label his 4 ties T1, T2, T3, and T4. Here is our diagram:



From our diagram, we can see that there are 12 different paths that we can take, each one representing a different combination of shirt and tie.

<u>ANSWER: There are 12 different combinations of shirts and ties that Mr. Mayer can make.</u>

Mr. Mayer's Shirts & Ties (continued)

Solution Strategy: Make a List

Another way to solve this problem is to make an organized list of all the different combinations. Again we'll label the shirts S1, S2, and S3, and we'll label the ties T1, T2, T3, and T4. Here is the list:

Shirt	Tie	Running Tally
S1	T1	1
S1	T2	2
S1	T3	3
S1	T4	4
S2	T1	5
S2	T2	6
S2	T3	7
S2	T4	8
S3	T1	9
S3	T2	10
S3	T3	11
S3	T4	12

Again, looking at our list, we see that there are 12 different combinations of shirts and ties.

ANSWER: There are 12 different combinations of shirts and ties that Mr. Mayer can make.

Planet Grumble & the 1,000-Day War

Name ______________________________ Date ____________

The Problem

There was a war on the planet Grumble. It lasted 1,000 days. If the planet Grumble uses the same calendar that we use on Earth, and the 1,000-day war started on a Monday, what day of the week did the 1,000-day war end?

Planet Grumble & the 1,000-Day War (continued)

Solution Strategy: Make a Simpler Problem and Make a Table

Let's put the information in a table (it could be considered a calendar) so it's easier for us to evaluate.   It's difficult for us to think about a number as large as 1,000.   Let's look at the beginning days of the war.

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6
Day 7	Day 8	Day 9	Day 10	Day 11	Day12	Day13
Day 14	Day 15	Day 16	Day 17	Day 18	Day 19	Day 20
Day 21	etc	etc	etc	etc	etc	etc

Solution Strategy: Look for a Pattern

What sort of pattern do we observe? We notice that all the multiples of 7 are landing on Sunday.   Let's find the closest multiple of 7 to 1,000.   We can do this by dividing 1,000 by 7.   We get 142.857.   We truncate the decimal portion and just use the whole number portion of our result.   Now we multiply 142 by 7 and we get 994. The closest multiple of 7 to 1,000 is 994.   We can check this by multiplying 7 by 143 and we get 1,001.   That's too much.

Let's redraw the chart or calendar showing the ending days of the war.

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
					Day 992	Day 993
Day 994	Day 995	Day 996	Day 997	Day 998	Day 999	Day 1,000

By making observations about a simpler problem, we were able to find a pattern that helped us solve the more difficult problem. The war ended on a Saturday.

ANSWER: The 1,000-day war on the planet Grumble ended on a Saturday.

Rectangular Patterns

The Problem

If you continue with the pattern shown below, what would the 20th figure look like?

Figure 1	Figure 2	Figure 3	Figure 4
o o	o o o o o o	o o o o o o o o o o o o	o o o o o o o o o o o o o o o o o o o o

Rectangular Patterns (continued)

<u>Solution Strategy: Look for a Pattern</u>

Figure 1	Figure 2	Figure 3	Figure 4
o o	o o o o o o	o o o o o o o o o o o o	o o o o o o o o o o o o o o o o o o o o

Let's compare the figures shown and look for a pattern.   How many dots are added each time?   To create Figure 2, 4 dots were added to the prior total. 2 + 4 = 6 total dots. To create Figure 3, 6 dots were added to the prior total. 6 + 6 = 12 total dots. To create Figure 4, 8 dots were added to the prior total. 12 + 8 = 20 total dots. Let's try the next 2 figures and see if the pattern is consistent.

<u>Figure 5</u>	**<u>Figure 6</u>**
o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o 10 dots were added. 20 + 10 = 30 total dots.	o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o 12 dots were added. 30 + 12 = 42 total dots.

It appears that the pattern is consistent.

Rectangular Patterns (continued)

Let's use a table to determine the number of dots in the 20th figure.

Figure Number	Number of Dots Added	Total Number of Dots
1	--	2
2	4	6
3	6	12
4	8	20
5	10	30
6	12	42
7	14	56
8	16	72
9	18	90
10	20	110
11	22	132
12	24	156
13	26	182
14	28	210
15	30	240
16	32	272
17	34	306
18	36	342
19	38	380
20	40	420

ANSWER: There are 420 dots in the 20th figure when this pattern is continued.

Restaurant Tables

Name ______________________________________ Date ________________

The Problem

A restaurant has a total of 60 tables. Some of the tables can seat 4 people and some can seat 2 people. All of the seats are occupied when there are 160 people seated. How many tables for 4 are there?

Restaurant Tables (continued)

Solution Strategy: Guess and Check

Let's start by randomly selecting 2 numbers whose sum is 60. That will represent the number of tables for 4 plus the number of tables for 2. Let's try 30 and 30.

Tables for 4 → 30 * 4 = 120 people
Tables for 2 → 30 * 2 = 60 people
120 + 60 = 180 people

That's too high. Let's try another guess. Since our first guess was too large, this time we need to make a guess that will result in less people seated in the restaurant. Let's lower the number representing the tables for 4. That will result in less people. Let's try 25 tables for 4 and 35 tables for 2.

Tables for 4 → 25 * 4 = 100 people
Tables for 2 → 35 * 2 = 70 people
100 + 70 = 170 people

That is still too high. Let's try lowering the number of tables for 4 even more. Let's try 20 tables for 4 and 40 tables for 2.

Tables for 4 → 20 * 4 = 80 people
Tables for 2 → 40 * 2 = 80 people
80 + 80 = 160 people

That's it!

ANSWER: There are 20 tables that seat 4 people and 40 tables that seat 2 people at the restaurant.

Sam the Soccer Man

Name ______________________________________ Date ________________

<u>The Problem</u>

Sam, the soccer man, has soccer balls for sale. He travels to the first soccer field and sells half of the soccer balls plus a half of a ball. Then he travels to the next soccer field and sells half of the remaining balls and half of a ball. He travels to a third soccer field and sells half of the remaining soccer balls and a half of a ball. Finally, he returns home with no soccer balls left. Sam accomplishes all of this without cutting any of the balls in half. How many soccer balls did Sam start with?

Sam the Soccer Man (continued)

Solution Strategy: Guess and Check

First of all, we need to deal with the issue of a "half of a ball". The story tells us that he does not cut any of the balls in half. How can this be? Well, we need to think about the problem in mathematical terms rather than visual terms. Let's begin by thinking about taking half of something. If we take half of an even number, we get a whole number with no fractional parts. But if we take half of an odd number, we get "something and a half". In this problem, Sam always sells half of the soccer balls plus a half of a ball. When we start with an odd number and take half, this will result in "something and a half". Then we add half, and we get a whole number. Aha! We're on to something!

Let's try making a guess. From the discussion above, we want to choose a number that is odd. We can also deduce that we need a number that is relatively small because there are only 3 fields for sales and Sam returns home with 0 soccer balls.

Let's try 15.

Sam starts with 15 soccer balls. At the first field he sells half of the soccer balls plus a half of a ball.

Half of 15 is 7.5 plus a half is 8. He sells 8 soccer balls at the first field.
That leaves him with 7 soccer balls (15 – 8 = 7).

That worked. We were able to take half and add a half without splitting up any of the soccer balls. Now Sam goes to the next field. He sells half of the remaining soccer balls plus a half of a ball.

Half of 7 is 3.5 plus a half is 4. He sells 4 soccer balls at the 2^{nd} field.
That leaves him with 3 soccer balls (7 – 4 = 3).

That also worked out okay. Now Sam goes to the third field with 3 soccer balls left. He sells half of them and a half of a ball.

Half of 3 is 1.5 plus a half is 2. He sells 2 soccer balls at the 3rd field.
That leaves him with 1 soccer ball (3 – 2 = 1).

Sam the Soccer Man (continued)

Now we have a problem. We were told that Sam returns home with NO soccer balls. Our guess of 15 resulted in Sam returning home with 1 soccer ball. This tells us that our guess must have been too high. What shall we try next?

<u>Let's try another odd number which is less than 15. Let's try 11 this time.</u>

<u>Field 1</u>: Half of 11 is 5.5, plus a half is 6 (11 – 6 = 5). He sells 6 soccer balls and is left with 5 soccer balls.

<u>Field 2</u>: Half of 5 is 2.5, plus a half is 3 (5 – 3 = 2). He sells 3 soccer balls and is left with 2.

<u>Field 3</u>: Half of 2 is 1, plus a half is 1.5. This will not work because we would need to cut a soccer ball in half to get 1.5. We are told in the problem that no soccer balls were cut in half.

<u>Let's try another odd number which is less than 11. Let's try 7 this time.</u>

<u>Field 1</u>: Half of 7 is 3.5, plus a half is 4 (7 – 4 = 3). He sells 4 and is left with 3.

<u>Field 2</u>: Half of 3 is 1.5, plus a half is 2 (3 – 2 = 1). He sells 2 and is left with 1.

<u>Field 3</u>: Half of 1 is .5, plus a half is 1 (1 – 1 = 0). He sells 1 and is left with 0.

That's it! We were told that he returns home with no soccer balls left. He must have started with 7 soccer balls for this to happen.

<u>ANSWER: Sam had 7 soccer balls to start with.</u>

Sam the Soccer Man (continued)

Solution Strategy: Work Backwards

We are given the ending piece of information in this problem. We are told that Sam returns home with 0 soccer balls. Remember that when you work backwards, you are doing the opposite – instead of subtracting, you add. Instead of taking half, you double. Let's work our way backward through the problem.

> **Field 3**: Sam sold half of his stock plus a half of a soccer ball. 0 plus a half is .5 Now we double it, .5 + .5 = 1. Sam must have arrived at Field 3 with 1 soccer ball.
>
> **Field 2**: Sam sold half of soccer balls plus a half of a soccer ball. 1 plus a half is 1.5. Now we double it, 1.5 + 1.5 = 3. Sam must have arrived at Field 2 with 3 soccer balls.
>
> **Field 1**: Sam sold half of the soccer balls plus a half of a soccer ball. 3 plus a half is 3.5. Now we double it, 3.5 + 3.5 = 7. Sam must have arrived at Field 1 with 7 soccer balls.

ANSWER: Sam had 7 soccer balls to start with.

Shape Equations

Name ______________________________ Date ____________

The Problem

In the 3 equations below, each of the shapes represents a number. Each shape represents the same number everywhere it occurs. What is the numerical value of the square?

□ + ○ + △ = 24

□ + □ + ○ = 20

□ + ○ + △ + △ = 34

Shape Equations (continued)

<u>Solution Strategy: Guess and Check</u>

□ + ○ + △ = 24

□ + □ + ○ = 20

□ + ○ + △ + △ = 34

Let's make some guesses about the values of the shapes. In the first equation we see that the sum is 24. We know that 24 ÷ 3 = 8. So let's make our first guess around 8. Let's try 7 for the square, 8 for the circle, and 9 for the triangle. If these are our values, the following equations must all be true:

7 + 8 + 9 should equal 24.	Yes. That is correct.
7 + 8 + 8 should equal 20.	No. It is equal to 23.

Let's try another guess around 8. Let's try 6 for the square, 8 for the circle, and 10 for the triangle.

6 + 8 + 10 should equal 24.	Yes. That is correct.
6 + 8 + 8 should equal 20.	Yes. That is correct.
6 + 8 + 10 + 10 should equal 34.	Yes. That is correct.

That's it! We have found all the values. The problem asks only for the value of the square and that is 6.

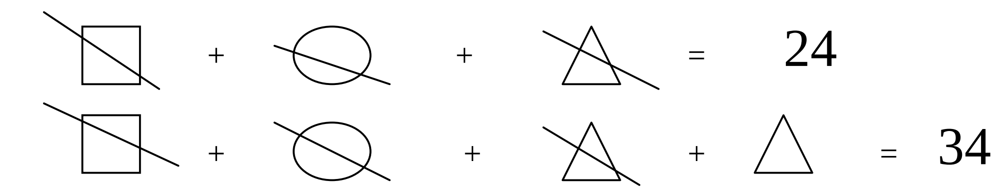
<u>ANSWER: The value of the square is 6.</u>

Shape Equations (continued)

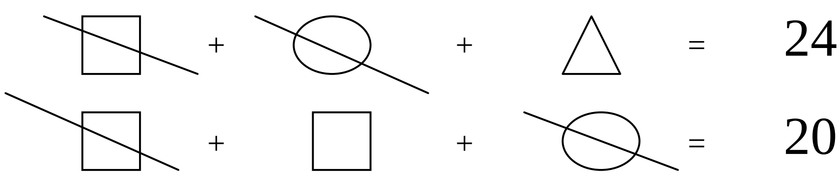
<u>Solution Strategy: Find a Pattern</u>

Let's analyze the 3 equations and see if we notice a pattern that might help us solve the problem. We can compare them and see how they are alike and how they are different.

For instance, the first equation and the last equation are the same EXCEPT for an additional triangle. Let's cross out the things that are the same. We are left with one triangle on the left of the equation. The sums of the 2 equations differ by 10 (34 – 24 = 10). Therefore, the triangle must be equal to 10. We can use this information to help us find the value of the square.



Let's look at the first 2 equations. How are they alike? How are they different? We notice that both equations have a square and a circle. If we eliminate those from the equations, we see that we have a triangle left in one equation and a square left in the other equation.



We see that the sums of the 2 equations differ by 4 (24 – 20 = 4) so we know that the square must be 4 less than the triangle. We know that the triangle is worth 10 from our prior findings. We now have enough information to find the value of the square
(10 – 4 = 6).

<u>ANSWER: The value of the square is 6.</u>

The Sliding Slug on the Slanted Sidewalk

Name ______________________________ Date ____________

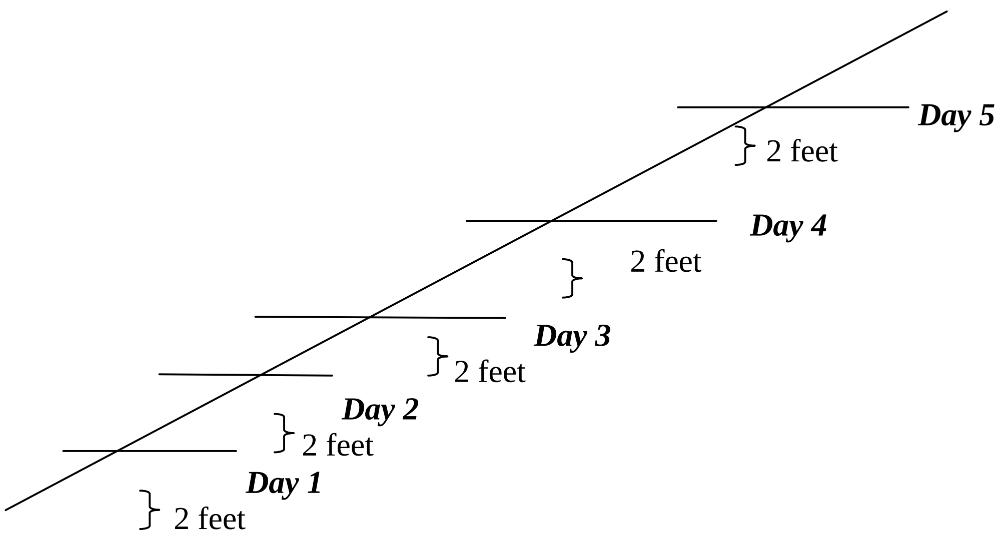
The Problem

A slug is at the bottom of a slanted sidewalk which is 10-feet long. The slug crawls 3-feet each day, but slides backwards 1-foot each evening. How many days will it take the slug to make its way to the end of the sidewalk?

The Sliding Slug on the Slanted Sidewalk (continued)

Solution Strategy: Draw a Picture

The best way to solve this problem might be to actually picture it. Let's draw a picture of the sidewalk and mark off the slug's travel progress. We'll advance the slug 2 feet each day since his daily progress is (3 – 1) or 2 feet.



2 + 2 + 2 + 2 + 2 = 10 feet.   It will take the slug 5 days to get to the top of the sidewalk.

ANSWER: It will take the slug 5 days to get to the end of the sidewalk.

Soccer Teams

Name ______________________________________ Date ________________

The Problem

A group of 4 soccer players, each from a different team, stopped at the same restaurant to eat after a tournament. They began to discuss how their teams did at the tournament. Determine what order their teams placed in the 4-team tournament (1^{st} through 4^{th} place) based on the discussion below. (Note: The players from the 1^{st} and 2^{nd} place team did not want to brag about their victories, so the statements they made are NOT truthful!)

Player A: "We did not come in last place."
Player B: "We did not come in 1^{st} place."
Player C: "We came in 3^{rd} place."
Player D: "My team did not come in 3^{rd} place."

Soccer Teams (continued)

<u>Solution Strategy: Make a Table</u>

Keeping in mind what each player said:

> Player A: "We did not come in last place."
> Player B: "We did not come in 1st place."
> Player C: "We came in 3rd place."
> Player D: "My team did not come in 3rd place."

Let's make a table to fill in our findings as we work our way through the problem. Our table will look like this:

Player Name	1st Place	2nd Place	3rd Place	4th Place
Player A				
Player B				
Player C				
Player D				

Player A states that his team did not come in last place. If he's lying, than his team DID come in last place. But that's contradictory because only the team members from the 1st or 2nd place team are lying. His team can't come in 1st or 2nd AND also come in 4th. He must be telling the truth (so he's not 1st or 2nd) and his team did not come in last place (so he's not 4th). By process of elimination, he must be on the 3rd place team. Let's mark that information in our table. Since his team is the 3rd place team, we can X that out for every other player.

Player Name	1st Place	2nd Place	3rd Place	4th Place
Player A	X	X	☺	X
Player B			X	
Player C			X	
Player D			X	

Player B states that his team did not come in 1st place. If that's true, then his team must have come in 2nd or 4th place. But it can't be 2nd place, because 2nd place team members are lying! If it's not true, then his team must have come in 1st place. We can eliminate the possibility of 2nd place for Player B but we still do not have enough information to determine if Player B's team was 1st or 4th. We'll leave it for now.

Player Name	1st Place	2nd Place	3rd Place	4th Place
Player A	X	X	☺	X
Player B		X	X	
Player C			X	
Player D			X	

Soccer Teams (continued)

Player C states that his team came in 3^{rd} place. We know this is a lie because Player A's team came in 3^{rd} place. So Player C must have come in 1^{st} or 2^{nd} place. We can now eliminate 3^{rd} and 4^{th} place for Player C but we still do not have enough information to determine whether Player C's team is 1^{st} or 2^{nd} place. We'll leave it for now.

Player Name	1^{st} Place	2^{nd} Place	3^{rd} Place	4^{th} Place
Player A	X	X	☺	X
Player B		X	X	
Player C			X	X
Player D			X	

Player D states that his team did not come in 3^{rd} place. We know that this is a true statement, so Player D's team cannot be 1^{st} or 2^{nd} (since he's not lying), and it must be 4^{th}. We can fill that in on our chart. Now that we know player D is in 4^{th} place, we can narrow down Player B to 1^{st} place. That leaves only 2^{nd} place for Player C. Our chart now looks like this:

Player Name	1^{st} Place	2^{nd} Place	3^{rd} Place	4^{th} Place
Player A	X	X	☺	X
Player B	☺	X	X	X
Player C	X	☺	X	X
Player D	X	X	X	☺

<u>ANSWER: The teams placed in the following order:</u>

1^{st} Place — Player B's team
2^{nd} Place — Player C's team
3^{rd} Place — Player A's team
4^{th} Place — Player D's team

Stacey's Schedule

Name ______________________________________ Date ________________

The Problem

River Wood Junior High School uses the following master schedule for scheduling each of the students in the school. The first 5 classes listed (including lunch) are mandatory; that is, everyone must take these classes. The last 4 classes are electives and students may choose 2 of these classes. An asterisk (*) indicates what periods the classes are available. Using the information in the master schedule, create a schedule for Stacey. She wants to take Keyboarding and Spanish as her electives. She is also involved in Student Council and they meet during the 1st lunch period. What will Stacey's schedule look like?

Class	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7
English		*		*			
Math	*		*			*	
Science		*		*		*	*
PE	*		*		*		*
Lunch				*	*		
Band		*					*
Spanish			*		*		
Keyboarding		*				*	
Shop			*		*	*	*

Stacey's Schedule

Solution Strategy: Make a Table

Here's the master schedule for the school:

Class	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7
English		*		*			
Math	*		*			*	
Science		*		*		*	*
PE	*		*		*		*
Lunch				*	*		
Band		*					*
Spanish			*		*		
Keyboarding		*				*	
Shop			*		*	*	*

Let's make a table that represents Stacey's schedule. It will look like this.

Period	Class
1	
2	
3	
4	
5	
6	
7	

Let's first see if any of Stacey's classes that MUST be taken during a particular period. Right away we need to note that Stacey MUST have lunch during 4th period because that is when Student Council meets. We also see that there are limited choices for Spanish and Keyboarding (only 2 classes available for each). Let's schedule these classes first. Let's put Spanish in 3rd period and Keyboarding in 2nd period.

Period	Class
1	
2	Keyboarding
3	Spanish
4	Lunch
5	
6	
7	

Stacey's Schedule (continued)

Now, let's look at the rest of the classes that Stacey needs to take. English is offered 2nd and 4th period. Uh oh. We have our first conflict. Lunch has to be 4th period, so let's see about shuffling her 2nd period class. Keyboarding is also offered 6th period so we'll move that there and put English in 2nd period.

Period	Class
1	
2	English
3	Spanish
4	Lunch
5	
6	Keyboarding
7	

Okay, so far, so good. Let's now take a look at Math. It is offered 1st, 3rd, or 6th period. We don't have anything scheduled in 1st period, so let's put it there.

Period	Class
1	Math
2	English
3	Spanish
4	Lunch
5	
6	Keyboarding
7	

That leaves Science and PE to be scheduled. We only have 2 periods left, 5th and 7th. Are these classes offered during that time? We look at the master schedule and see that PE is available 5th period and Science is available 7th period. It works!

ANSWER: Here is Stacey's final schedule.

Period	Class
1	Math
2	English
3	Spanish
4	Lunch
5	PE
6	Keyboarding
7	Science

Tennis Tournament

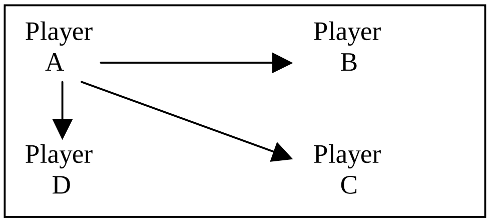
Name ____ Date ________________

The Problem

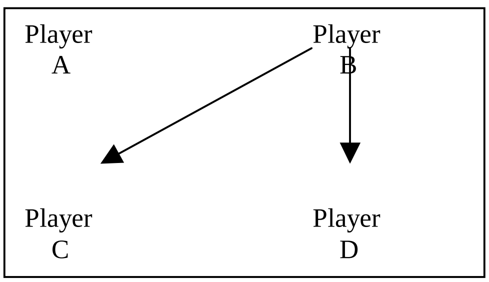
There are 4 people in a singles tennis tournament. If each player only plays one match with each other player, how many matches are played in all?

Tennis Tournament (continued)

<u>Solution Strategy: Draw a Picture</u>

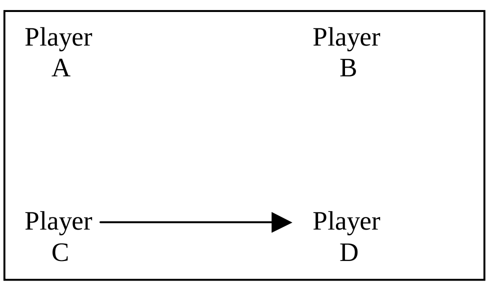


Player A plays Player B; that's 1 match.
Player A plays Player C; that's 2 matches.
Player A plays Player D; that's 3 matches.



Player B plays Player C; that's 4 matches.
Player B plays Player D; that's 5 matches.

Notice that we didn't include the matches between Player A and Player B because we already counted them in Player A's matches.



Player C plays Player D; that's 6 matches.

Notice again that we didn't include prior matches between Player A and Player C, and matches between Player B and Player C.

We have previously listed all the matches for Player D with Players A, B, and C, so we are finished.   The total is 6 matches.

<u>ANSWER: There will be a total of 6 matches played in the tournament if each of the 4 players only plays each other once.</u>

Tennis Tournament (continued)

Solution Strategy: Make a List

Match Number	Player	Opponent
1	A	B
2	A	C
3	A	D
4	B	C
5	B	D
6	C	D

ANSWER: There will be a total of 6 matches played in the tournament if each of the 4 players only plays each other once.

Solution Strategy: Make a Simpler Problem and Look for a Pattern

What if there was only one player?
There would be 0 matches because you can't play a match with only one player.

What if there were 2 players?
Player A → Player B
There would be 1 match between 2 players.

What if there are 3 players?
Player A → Player B
Player A → Player C
Player B → Player C
There would be 3 matches.

What if there are 4 players?
Player A → Player B
Player A → Player C
Player A → Player D
Player B → Player C
Player B → Player D
Player C → Player D
There would be 6 matches.

I'm starting to see a pattern.   It's a stair-step pattern. When there is one player, there are 0 matches.   When there are 2 players, there is (0 + 1) or 1 match played.   When there are 3 players, there are (0 + 1 + 2) or 3 matches played.   When there are 4 players, there are (0 + 1 + 2 + 3) or 6 matches played.

Note to Tutor/Teacher:
For a more difficult problem, increase the number of players in the tournament.   If each player only plays each other once and there are ***n*** players in the tournament, the solution will always be 1 + 2 + ….. + ***n-1*** .

Tommy's 12-Hour Clock

Name ______________________________________ Date ________________

The Problem

Tommy has a 12-hour clock. The clock runs continuously without stopping. It is currently 2:00 p.m. What time will be shown on Tommy's clock 1,000 hours from now? Will it be a.m. or p.m.?

Tommy's 12-Hour Clock (continued)

<u>Solution Strategy: Make a Simpler Problem</u>

It is difficult to think about 1,000 hours! Let's look at a simpler problem. Let's break apart the 1,000 hours into 12-hour intervals. We'll examine the first few to see if there is a pattern that shows up that will help us to solve the larger problem.

Interval Number (12-hr interval)	Current Time	a.m. or p.m.?
Starting Time	2:00	p.m.
1	2:00	a.m.
2	2:00	p.m.
3	2:00	a.m.
4	2:00	p.m.
5	2:00	a.m.

We begin to see a pattern by looking at 12-hour intervals. We see that the time is the same, 2:00, and the only thing that changes or alternates is the a.m. and p.m. We see a pattern that on odd-numbered intervals, it is a.m., and on even-numbered intervals, it is p.m. We have found some useful patterns.

How can this help us with the bigger problem? Well, we now know that it is helpful to break the larger time into 12-hour intervals. We might ask ourselves, "how many complete 12-hour intervals are there in 1,000 hours?"

Our number model will look like this: 1,000 ÷ 12 = 83.3333

We just need to look at the whole number portion. We know there are 83 of the12-hour intervals in 1,000 hours. We then ask ourselves, "how many hours are left over?"

Our number models will look like this: 12 * 83 = 996 hours
1,000 – 996 = 4 hours left over

Since the time at each 12-hour interval is 2:00, we add the 4 hours to that. 4 hours after 2:00 is 6:00. But how do we know if it's a.m. or p.m.? Well, we discovered earlier in the problem that each odd-numbered interval is a.m. and each even-numbered interval is p.m. The 83rd interval is an odd-numbered interval; therefore, it is a.m.

<u>ANSWER: The time shown on Tommy's clock after 1,000 hours is 6:00 a.m.</u>

Triangular Patterns of Circles

Name ______________________________________ Date ________________

<u>The Problem</u>

If you continue with the pattern shown below, what would the 8th figure look like?

Figure 1	Figure 2	Figure 3
o o o	o o o o o o	o o o o o o o o o o

Triangular Patterns of Circles (continued)

<u>Solution Strategy: Look for a Pattern</u>

Figure 1	Figure 2	Figure 3
o o o	o o o o o o	o o o o o o o o o o

Let's first discuss what we observe about figures 1, 2, and 3. We notice that in Figure 2 we add 3 more dots to get a total of 6 dots. Then in Figure 3, we add 4 more dots to get a total of 6 + 4 or 10 dots.

What would Figure 4 look like? Let's draw it.

o

o o

o o o

o o o o

o o o o o

We see in Figure 4 that we add 5 dots and get a total of 10 + 5 or 15 dots.
Let's enter our data in a chart and examine it more carefully.

Figure Number	Number of dots to start with	Number of dots added	Total number of dots
1	3	--	3
2	3	3	6
3	6	4	10
4	10	5	15

Triangular Patterns of Circles (continued)

What kind of a pattern do we see? We see that in each figure, our total number of dots increases by the next consecutive integer. First 3 more, then 4 more, then 5 more, etc.

How many dots will the 8th figure have? Let's expand our chart to show the 8th figure.

Figure Number	Number of dots to start with	Number of dots added	Total number of dots
5	15	6	21
6	21	7	28
7	28	8	36
8	36	9	45

ANSWER: There are 45 dots in the 8th figure if you continue with this pattern.

What Number Am I?

Name ______________________________________ Date ________________

The Problem

A palindrome is a number or a word that is read the same forward or backward. A few examples of words that are palindromes are: mom, dad, and racecar. A few examples of numbers that are palindromes are: 11, 202, and 41014.

Find the following number. I am a palindrome. I am between 50,000 and 60,000. I am evenly divisible by 9. The number in my hundreds place is 0. What number am I?

What Number Am I? (continued)

Solution Strategy: Make a Chart

We need to carefully lay out each digit of the number we are looking for. We know that it is a 5-digit number and that it starts with a 5 (we know this because it's between 50,000 and 60,000).

5				

We also know that it ends with a 5 (because it is a palindrome).   We know that the number in the hundreds place is a 0 (given in the problem).

5		0		5

What do we know about the remaining missing numbers? Well, we know that the 2 missing numbers are the same (because this is a palindrome).

We know that the number is divisible by 9 (given in the problem). We need to think about what it means for a number to be divisible by 9. The divisibility rule for 9 is that if the sum of the digits is divisible by 9, then the number is divisible by 9. Let's add up the digits we have thus far. 5 + 0 + 5 = 10.

What number when doubled can I add to 10 and have it result in a number that is divisible by 9?

What Number Am I? (continued)

Let's try all of the possibilities.

Number	Sum of the digits	Divisible by 9? Yes or No
50,005	5 + 0 + 0 + 0 + 5 = 10	No
51,015	5 + 1 + 0 + 1 + 5 = 12	No
52,025	5 + 2 + 0 + 2 + 5 = 14	No
53,035	5 + 3 + 0 + 3 + 5 = 16	No
54,045	5 + 4 + 0 + 4 + 5 = 18	Yes
55,055	5 + 5 + 0 + 5 + 5 = 20	No
56,065	5 + 6 + 0 + 6 + 5 = 22	No
57,075	5 + 7 + 0 + 7 + 5 = 24	No
58,085	5 + 8 + 0 + 8 + 5 = 26	No
59,095	5 + 9 + 0 + 9 + 5 = 28	No

The only number that meets the divisibility by 9 rule is 54,045.

ANSWER: The number is 54,045.

Work Schedules for Sammy & Elyse

Name ________________________________ Date ______________

The Problem

Sammy and Elyse work at the same grocery store. They are both part-time employees. Elyse is always scheduled to work 1 day and then has 2 days off. Sammy works 1 day and then has 3 days off. The store is open every day. If Elyse and Sammy both worked on Wednesday, when is the next time they will both be scheduled on the same day?

Work Schedules for Sammy & Elyse (continued)

Solution Strategy: Make a Table

Let's put the information in a table (it could be considered a calendar or a schedule) so it's easier for us to evaluate. We'll start both employees on Wednesday. Then we will fill in the following days according to the schedule given above. That is, Elyse works 1 day and is off 2 days. Sammy works 1 day and is off 3 days. We will do this until both employees are scheduled again on the same day.

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
			Elyse Sammy			Elyse
Sammy		Elyse		Sammy	Elyse	
	Elyse Sammy					

ANSWER: It will be 12 days before Sammy and Elyse are schedule to work on the same day again.

XYZ Equation

Name ________________________________ Date ______________

The Problem

In the following equation, the sum of two 3-digit numbers is represented. The variables X, Y, and Z represent the digits 7, 8, and 9, but not necessarily in that order. Each digit can only be used once. What is the largest value the sum can be?

$$\begin{array}{r} X\ Y\ Z \\ +\quad Z\ Y\ X \\ \hline \end{array}$$

XYZ Equation (continued)

Solution Strategy: Make a Table

```
     X Y Z
+    Z Y X
```

Let's make a table to show all the possible 3-digit number combinations that we can make. The table will help us organize our work.

X Y Z	Z Y X	Sum of the 2 numbers
789	987	789 + 987 =1,776
798	897	798 + 897 =1,695
879	978	879 + 978 =1,857
897	798	897 + 798 =1,695
978	879	978 + 879 =1,857
987	789	987 + 789 =1,776

We see from our table that the largest sum that can be made from all the possible 3-digit number combinations is 1,857.

ANSWER: The largest sum that can be made is 1,857.